

# Numerical Modeling of Optical Fibers Using the Finite Element Method and an Exact Non-reflecting Boundary Condition

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## Abstract

© 2018 Walter de Gruyter GmbH, Berlin/Boston. The original problem for eigenwaves of weakly guiding optical fibers formulated on the plane is reduced to a convenient for numerical solution linear parametric eigenvalue problem posed in a disk. The study of the solvability of this problem is based on the spectral theory of compact self-adjoint operators. Properties of dispersion curves are investigated for the new formulation of the problem. An efficient numerical method based on FEM approximations is developed. Error estimates for approximate solutions are derived. The rate of convergence for the presented algorithm is investigated numerically.

<http://dx.doi.org/10.1515/cmam-2017-0049>

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## Keywords

Exact Non-Reflecting Boundary Condition, Finite Element Method, Optical Fiber, Spectral Problem

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